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Universal Torsion–Induced Interaction from Large Extra Dimensions

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Abstract

We consider the Kaluza–Klein (KK) scenario in which only gravity exists in the bulk. Without the *assumption* of symmetric connection, the presence of brane fermions induces torsion. The result is a universal axial contact interaction that dominates those induced by KK gravitons. This enhancement arises from a large spin density on the brane. Using a global fit to Z–pole observables, we find the 3σ bound on the scale of quantum gravity to be 28 TeV for $n = 2$. If Dirac or light sterile neutrinos are present, the data from SN1987A increase the bound to $\sqrt{n}M_S \geq 210$ TeV.

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Consistent string theories require dimensions beyond the usual four, that could in principle be as large as a millimeter. These large extra dimensions provide new avenues for solution to the hierarchy problem [1] and have significant phenomenological consequences as well. Current collider experiments set a lower bound of about 1 TeV on the fundamental Planck scale, while cosmological considerations require it to be around 50 TeV for two extra dimensions. Future colliders will be able to probe up to 5∼8 TeV [2].

This paper considers the minimal Kaluza–Klein scenario (KK) wherein only gravity exists in the bulk while all Standard Model fields are localized on a 4D brane. Without *a priori* assumptions on its symmetry, fermions always induce antisymmetric pieces, or torsion, in the gravity connection. We show that this implies a universal $U(45)$–invariant contact interaction which is suppressed only by the square of the *fundamental* Planck scale. This interaction therefore dominates four–fermion interactions induced by KK graviton exchange at current collider energies and provides important information concerning the viability of extra dimensions. The enhancement is distinct from the KK mechanism. It originates from a large $4+n$ dimensional spin density due to the spinor fields confined to a 4 dimensional brane. We obtain a limit on this interaction from electroweak precision data and compare it to other constraints available from particle physics and astrophysics.

While the metric is coupled to the energy–momentum tensor in gravity, torsion is coupled directly to the spin density of matter systems [3,4]. It appears in any description of gravity where Lorentz transformations are treated as local symmetries and is a feature of string theory and other variations of general relativity. The basic quantity is the torsion tensor $T^{\alpha}{}_{\beta\gamma}$, defined as the antisymmetric part of the connection $\tilde{\Gamma}^{\alpha}{}_{\beta\gamma}$:

$$T^{\alpha}{}_{\beta\gamma} = \tilde{\Gamma}^{\alpha}{}_{\beta\gamma} - \tilde{\Gamma}^{\alpha}{}_{\gamma\beta}. \tag{1}$$

Torsion violates the equivalence principle in its *very strong* form [5], as it cannot be removed by an appropriate choice of coordinates. It does not directly affect the propagation of light and test particles, and thus cannot be probed by standard tests of general relativity [5].

We denote with a tilde quantities derived from a non–symmetric connection $\tilde{\Gamma}^{\alpha}{}_{\beta\gamma}$, while those without a tilde refer to quantities derived from Christoffel symbols $\Gamma^{\alpha}{}_{\beta\gamma}$.[1] Via the metric condition $\tilde{\nabla}_{\alpha} g_{\mu\nu} = 0$, the torsion tensor can be related to the contorsion tensor $K^{\alpha}{}_{\beta\gamma}$ defined by

$$\tilde{\Gamma}^{\alpha}{}_{\beta\gamma} = \Gamma^{\alpha}{}_{\beta\gamma} + K^{\alpha}{}_{\beta\gamma}, \tag{2}$$

such that $K_{\alpha\beta\gamma} = \frac{1}{2}\left(T_{\alpha\beta\gamma} - T_{\beta\alpha\gamma} - T_{\gamma\alpha\beta}\right)$. $T^{\alpha}{}_{\beta\gamma}$ in 4D contains 24 independent components. However, only its totally antisymmetric part, expressible as an axial vector S^{σ},

$$S^{\sigma} = i\, \epsilon^{\mu\nu\rho\sigma} T_{\mu\nu\rho} \tag{3}$$

is relevant for spin 1/2 fermions [6]. In what follows we assume that torsion is completely antisymmetric. The relations above (except for Eq. (3)) can be generalized for the case of $4+n$ dimensions in a straightforward manner. We will use the following convention for the

[1] We follow the conventions and definitions of Ref. [6] except for the definition of γ_5.

indices [8]: $\mu = 1, \cdots, 4$, $\hat{\mu} = 1, \cdots, 4+n$, $i = 5, \cdots, 4+n$. The signature of the metric is $(1, -1, \cdots, -1)$.

To begin, note that torsion minimally couples to fermions only [6]. The minimal action for $4+n$ dimensional gravity coupled to fermions localized on a 4–dimensional brane is:

$$
\begin{aligned}
S = &-\frac{1}{\hat{\kappa}^2}\int d^{4+n}x\,\sqrt{|\hat{g}_{4+n}|}\;\tilde{R} \\
&+\int d^4x\,\sqrt{|\hat{g}_4|}\;\frac{i}{2}\left[\bar{\Psi}\gamma^\mu\tilde{\nabla}_\mu\Psi-\left(\tilde{\nabla}_\mu\bar{\Psi}\right)\gamma^\mu\Psi+2iM\bar{\Psi}\Psi\right],
\end{aligned}
\tag{4}
$$

Here $\hat{\kappa}^2 = 16\pi G_N^{(4+n)}$, $\tilde{R}$ is the $4+n$ dimensional scalar curvature, and $\hat{g}_{4+n}$ and $\hat{g}_4$ are respectively the $4+n$ and 4–dimensional (induced) metric determinants.

The covariant derivative $\tilde{\nabla}_\mu$ is defined by $\tilde{\nabla}_\mu\Psi = \partial_\mu\Psi + \frac{i}{2}\tilde{\omega}_\mu^{ab}\sigma_{ab}\Psi$, where $\tilde{\omega}_\mu^{ab}$ is the spin–connection, $\sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b]$, with a, b the local Lorentz indices. A general spin connection $\tilde{\omega}_\mu^{ab}$ can be expressed in terms of a torsion-free spin-connection ω_μ^{ab}, the contorsion tensor and the vierbein e_μ^a:

$$
\tilde{\omega}_\mu^{ab} = \omega_\mu^{ab} + \frac{1}{4}K^\nu{}_{\lambda\mu}\left(e^{\lambda a}e_\nu^b - e^{\lambda b}e_\nu^a\right). \tag{5}
$$

Upon substitution, the action in the case of completely antisymmetric torsion becomes

$$
\begin{aligned}
S = &-\frac{1}{\hat{\kappa}^2}\int d^{4+n}x\,\sqrt{|\hat{g}_{4+n}|}\;\left(R - K^{\hat{\mu}\hat{\nu}\hat{\rho}}K_{\hat{\mu}\hat{\nu}\hat{\rho}}\right) \\
&+\int d^4x\,\sqrt{|\hat{g}_4|}\;i\bar{\Psi}\left(\gamma^\mu\nabla_\mu - \frac{1}{8}S^\mu\gamma_\mu\gamma_5 + iM\right)\Psi\,.
\end{aligned}
\tag{6}
$$

Here R is the $4+n$ dimensional *metric* curvature, $K^{\hat{\mu}\hat{\nu}\hat{\rho}} = \frac{1}{2}\,T^{\hat{\mu}\hat{\nu}\hat{\rho}}$, and ∇_μ is the conventional covariant derivative without torsion. We use $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$. The resultant equations of motion are

$$
\begin{aligned}
&K_{i\hat{\mu}\hat{\nu}} = 0\,, \\
&S_\mu = i\frac{3}{2}\frac{\sqrt{|\hat{g}_4|}}{\sqrt{|\hat{g}_{4+n}|}}\;\hat{\kappa}^2\;\bar{\Psi}\gamma_\mu\gamma_5\Psi\;\delta^{(n)}(x)\,, \\
&R_{\hat{\mu}\hat{\nu}} - \frac{1}{2}g_{\hat{\mu}\hat{\nu}}R = \frac{\hat{\kappa}^2}{2}\;T_{\hat{\mu}\hat{\nu}} + \mathcal{O}\left(\hat{\kappa}^4\right)\,.
\end{aligned}
\tag{7}
$$

$T_{\hat{\mu}\hat{\nu}}$ is the torsion–free energy–momentum tensor for the matter fields. The first two of these relations are algebraic constraints. Classically, torsion does not propagate and is zero outside the matter distribution [4]. Its source is the *spin density* of fermions confined on the brane. Elimination of S_μ from the action via Eq. (7) produces a fermion contact interaction:

$$
\begin{aligned}
S = &-\frac{1}{\hat{\kappa}^2}\int d^{4+n}x\,\sqrt{|\hat{g}_{4+n}|}\;R \\
&+\int d^4x\,\sqrt{|\hat{g}_4|}\;\left[\bar{\Psi}\left(i\gamma^\mu\nabla_\mu - M\right)\Psi + \frac{3}{32}\frac{\sqrt{|\hat{g}_4|}}{\sqrt{|\hat{g}_{4+n}|}}\hat{\kappa}^2\;\left(\bar{\Psi}\gamma_\mu\gamma_5\Psi\right)^2\;\delta^{(n)}(0)\right].
\end{aligned}
\tag{8}
$$

The delta–function appearing in this expression should be regularized to account for a finite brane width:

$$\delta^{(n)}(0) \to \frac{1}{(2\pi)^n}\int_0^{M_S} d^n k = \frac{M_S^n}{2^{n-1}\pi^{n/2}\, n\,\Gamma\left(\frac{n}{2}\right)} \ , \tag{9}$$

M_S is the cutoff scale of the effective theory, here taken to be of the order of the inverse brane width. The $4+n$ dimensional coupling constant $\hat{\kappa}$ is related to the 4–dimensional coupling κ and the volume of the extra dimensions compactified on a torus via $\hat{\kappa}^2 = \kappa^2 V_n = 16\pi(4\pi)^{n/2}\Gamma(n/2)M_S^{-(n+2)}$ [8][2]. As a result, the leading $\mathcal{O}\left(\hat{\kappa}^2\right)$ torsion contribution to the action is given by

$$\Delta S = \int d^4x\, \frac{3\pi}{nM_S^2}\left[\sum_j \bar{\Psi}_j\gamma_\mu\gamma_5\Psi_j\right]^2 \ , \tag{10}$$

where j runs over all fermions existing on the brane. The expansion in $\hat{\kappa}$ is expected to be valid provided the typical energy E of a physical process is below the cutoff scale M_S (see also Ref. [9]). In this case a typical "size" of torsion is considerably below E. Note that the exact coefficient in Eq.(10) depends on the assumptions about the short-distance physics; in particular, it depends on the regularization of the delta-function.

The interaction (10) is the unique contact interaction possessing the maximal approximate global symmetry of the minimal Standard Model, *i.e.* the group $U(45)$ acting on the 45 Weyl spinors $\Psi_L = (q_L, u_R^c, d_R^c, l_L, e_R^c)$ [10][3]. The effect is truly universal for all fermions, in contrast to the four–fermion operators induced by KK graviton exchanges. Graviton couplings are mass and energy dependent, leading to different strengths for different fields. Finally, the KK–induced interactions have two additional suppression factors: s/M_S^2 [8] and f^2/M_S^2 [11]. The former follows from the graviton coupling to the energy–momentum tensor, the latter from the brane recoil effects (note that the rigidity of the brane f plays no role in our argument). Consequently, at typical accelerator energies the interaction (10) is enhanced over the KK–induced interactions by *orders of magnitude* (*e.g.* at LEP energies the enhancement factor is about 10^2) and will completely dominate. This enhancement results from a large $4+n$ dimensional *spin density* on the brane and is present whenever fermions are localized. Note that the interaction (10) is repulsive for aligned spins [4].

Let us briefly discuss how this result is modified by quantum corrections [7]. Generally, fermion loops will induce propagation of torsion along the brane, with $S_\mu\bar{\Psi}\gamma^\mu\gamma_5\Psi$ generating the relevant kinetic terms. The associated quantum of torsion will have a mass of order M_S. Its propagation effects are therefore irrelevant at typical accelerator energies. Loop corrections to the tree level torsion coupling and mass evaluated using an explicit cutoff amount to a rescaling of M_S in Eq. (10) by a factor of 1∼2. However, if we use dimensional

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[2]For simplicity we set the string scale and the $4+n$ dimensional Planck mass equal.

[3]Other contact interactions possessing the same symmetry can be brought into the form of Eq. (10).

regularization, the result is largely insensitive to radiative corrections due to the absence of quadratic divergences.

The universal interaction (10) will affect Z–pole electroweak observables. Corrections to the oblique parameters appear at the two–loop level and can be neglected [4]. Two types of vertex corrections are shown in Fig. 1. The combined contribution of the diagram in Fig. 1a and the corresponding wave function renormalization diagrams is suppressed by $1/M_S^2$ and leads only to a *universal* multiplicative correction to the couplings (neglecting light fermion masses). Since the observables we will consider are ratios of couplings, such corrections will cancel. The diagram in Fig. 1b is significant. Note that the corresponding wave function renormalization diagram vanishes. Summing contributions of all of the fermions and taking into account $\sum I_3 = 0$, we write the leading correction to the Z–couplings as

$$\delta h_L = -\delta h_R = \frac{3N_c m_t^2}{4\pi n M_S^2}\ \ln \frac{M_S^2}{m_t^2}\ , \tag{11}$$

where the Z–vertex is defined as $-i\frac{g}{\cos\theta_W} Z_\mu \bar{\Psi}\gamma^\mu (P_L h_L + P_R h_R)\Psi$. The contribution of torsion to δh_L is strictly positive.

We perform a global fit to the LEP/SLD electroweak observables including $R_{\nu/\ell} = \Gamma(Z \to \nu\bar{\nu})/\Gamma(Z \to \ell^+\ell^-)$, $R_{b,c}$, $A_{FB}(i)$ and A_i $(i = e, \mu, \tau, b, c)$ [5]. The data used and a detailed description of the technique (applied to a different model) can be found in [12]. Note that the KK graviton radiative corrections are suppressed as discussed above; moreover, the KK vertex corrections will largely cancel in our set of observables since, for the case of light fermions, they modify the couplings multiplicatively. The KK graviton corrections to the oblique parameters may not be negligible [13] and can affect our fit results through $\sin^2\theta_W$. In the fit, we leave $\delta s^2 \equiv \sin^2\theta_W - [\sin^2\theta_W]_{\rm SM}$ as a free parameter to account for a variation in the Higgs mass and KK graviton radiative corrections. From a two–parameter fit we obtain

$$\begin{aligned} \delta h_L &= -0.00049 \pm 0.00021 \\ \delta s^2 &= -0.00068 \pm 0.00018\ . \end{aligned} \tag{12}$$

The $\chi^2/d.o.f.$ of the fit is 17/12. Fig. 2 shows that δh_L is most strongly constrained by $R_{\nu/\ell}$. Since the experimental value for $R_{\nu/\ell}$ is about 2σ below the SM prediction, the preferred value of δh_L is about 2σ below zero. The value of δh_L is almost uncorrelated with δs^2 and thus with the Higgs mass. As a result of this classical statistical analysis, the model is excluded at the 2σ level since it generates only a positive δh_L. Using Eq. (11), we obtain the 3σ bound on M_S:

$$M_S \geq 28\ \text{TeV} \tag{13}$$

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[4]Throughout this analysis we retain only leading tree or one–loop contributions. This approximation is valid to leading order in $1/n$.

[5]We omit R_ℓ from our fit since a universal correction to R_ℓ only shifts the value of $\alpha_s(M_Z)$.

for $n = 2$. For $n = 4(6)$ the bound weakens to 19 (15) TeV. This implies that we do not expect deviations from Newton's law at distances above 9×10^{-4} mm for $n = 2$.

We next consider other constraints on the universal interaction[6]. $A \times A^+$ contact interaction (10) affects at the tree level the differential cross sections for $e^+e^- \to f\bar{f}$ measured at LEP. The OPAL measurements [15] imply

$$\sqrt{n}M_S \geq 10.3 \text{ TeV} \tag{14}$$

at the 95% confidence level. Electron–quark contact interactions can also be constrained via HERA DIS data, Drell–Yan production at the Tevatron, *etc.* The global analysis [16] yields

$$\sqrt{n}M_S \geq 5.3 \text{ TeV} \ . \tag{15}$$

Another potentially strong constraint can come from the measurement of the invisible width of the Υ and J/Ψ resonances at B and τ–charm factories [17].

A powerful astrophysical constraint can be derived if we admit existence of Dirac or light sterile neutrinos. For the case of Dirac neutrinos, the torsion–induced interaction containes a term

$$\Delta\mathcal{L} = -\frac{6\pi}{nM_S^2}\ \bar{q}\gamma^\mu\gamma_5 q\ \bar{\nu}_R\gamma_\mu\nu_R \ . \tag{16}$$

This contact interaction provides a new channel of energy drain during neutron star collapse, since right handed neutrinos produced by nucleon interactions leave the core without rescattering [18]. This would affect neutron star evolution; in particular, it would modify the duration of the standard neutrino burst. From observations of SN 1987A one infers [18]

$$\sqrt{n}M_S \geq 210 \text{ TeV} \ . \tag{17}$$

Similar considerations apply to the case of light sterile neutrinos: if $m_{\nu_s} \ll 50$ MeV, the core temperature, the analysis is completely analogous to that of Dirac neutrinos and the bound (17) holds. This bound translates into an upper bound on the compactification radius of 3×10^{-5} mm for $n = 2$.

Since M_S controls all gravity effects in extra dimensions, the limits on M_S being larger than tens of TeV reported here imply weaker KK graviton couplings than those considered in the literature. The limits were obtained under a minimal set of assumptions in the context of physics of extra dimensions. We first consider the general set of connections consistent with general covariance and local Lorentz symmetries. We fix all matter fields to be on a brane, consistent with the most conservative scenario. The consequence is that tree–level effects from a minimal action are sufficiently strong to produce the bounds reported above. Even for less conservative scenarios the universal interaction (10) can be expected to dominate possible KK gauge effects as long as the fermions are confined to the brane. We emphasize that this interaction is generic unless we impose the additional condition that the connection

[6]Phenomenological implications of 4–d gravity with torsion were also considered in [14].

be symmetric.[7] Finally, it is interesting to note that, in this context, particle physics places a constraint on violation of the strong equivalence principle.

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[7] The analogous four–fermion interaction involving gauginos in supergravity models needs a separate discussion because of their connections to the corresponding coupling among gauge bosons. However these terms do not affect the kinds of phenomenology addressed in the present paper.

FIGURES

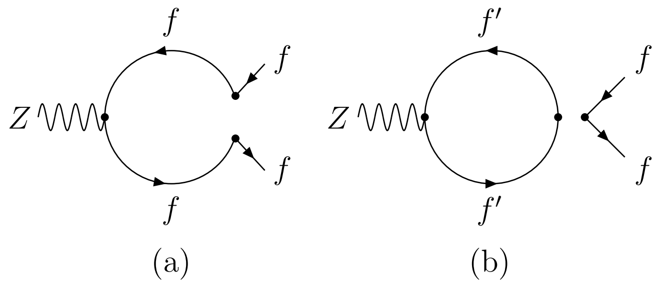

FIG. 1. Corrections to the $Zf\bar{f}$ coupling from the universal contact interaction Eq. (10).

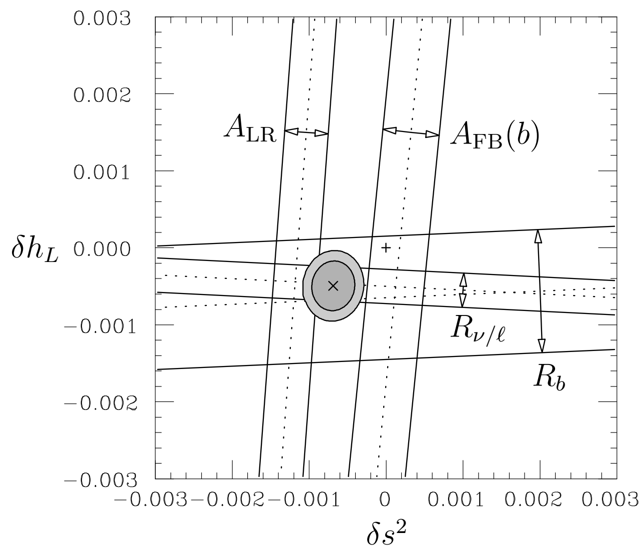

FIG. 2. The 68% and 90% confidence contours in the δs^2–δh_L plane. The 1σ bounds from the observables leading to the strongest constraints are also shown.

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